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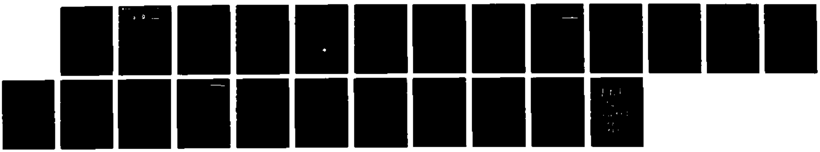

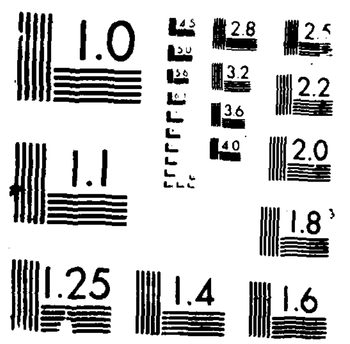
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The research accomplishments of three principal investigators are described. Areas with results are system reliability, combination of opinions, Bayesian applications to data analysis and quality assurance, reliability growth and software reliability, Bayesian approximation methods, risk portfolio problems, hierarchical models, simulation, estimation and testing, reliability models, and peaks from random data.

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RESEARCH OF RICHARD E. BARLOW

I will describe my research progress and significant results in terms of three areas of research interest; namely 1) System reliability, 2) Combination of opinions; and 3) Bayesian statistical applications to data analysis and quality assurance.

1. System Reliability

Perhaps the most important contribution was the generalization and simplified proof of the signed domination theorem [cf. Set Theoretic Signed Domination for Coherent Systems (1982) and Computational Complexity of Coherent Systems and the Reliability Polynomial (1985)]. The signed domination theorem lies at the heart of the proof of the topological formula and many other key results in network reliability theory. It is a unifying result, some of whose applications were also described in "A Survey of Network Reliability and Domination Theory" (1984).

In system reliability prediction, one of the most difficult problems (especially if the classical statistics approach is used) is to combine component and system failure data. A Bayesian approach based on calculating the posterior variance is described in "Combining Component and System Information in System Reliability Calculation" (1985) and also in "Assessing the Reliability of Computer Software..." (1985).

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2. Combining Expert Opinions

Two important research questions were addressed in different publications. The first question is, How should a decision maker combine the opinions from several experts about an unknown quantity? In "Combination of Experts' Opinions Based on Decision Theory" (1986) an approach was suggested for the case when it is not appropriate for the decision maker to exercise more than minimal judgement as in the case of a government agency. The second question is, How should a group reach a consensus relative to an unknown quantity, based on their possibly very different opinions? The group Pareto optimal decisions are characterized in "The Group Consensus Problem" (1985). A result concerning Pareto optimal group decisions (which was mistakenly attributed to de Finetti) is refined and generalized. It turns out that de Finetti's paper (which was in Italian) actually contains a different set of results. The translation and investigation of the implications of de Finetti's important results are still being pursued.

3. Bayesian Statistical Applications

The Bayesian approach is used in "A Critique of Deming's Discussion of Acceptance Sampling Procedures" (1986) to correct Deming's rule that in inspection sampling the only rule which should be followed is the "all or none" inspection rule. This is true if the percentage defective in a lot is a priori fairly well specified but not if there is sufficient initial uncertainty. Computing algorithms are given together with elegant analytical solutions for special cases.

In "Informative Stopping Rules" (1984) the case when the stopping rule is informative relative to some examples which arose in practice is examined in detail. This is important because almost all models in the literature assume that the stopping rule is noninformative.

A recent paper "Using Influence Diagrams to Solve the Calibration Problem" considers the problem of designing an experiment to calibrate a measuring instrument. A numerical algorithmic solution is provided for the case when the prior distributions are multivariate normal. The number of required integrations is reduced to three. This means the problem can be solved on a desktop computer. In general the problem will require a much larger computer. From a theoretical standpoint, the most interesting result is that, unlike the usual linear regression experimental design problem, the optimal design in the inverse linear regression problem does not, in general, lie on the boundary of the feasible region.

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2. Combining Expert Opinions

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Papers by R. E. Barlow and co-authors appearing in this volume:

- Mathematical theory of reliability. Historical Perspectives, pp. 3-11.
- Distributions with monotone failure rate (with F. Proschan), pp. 12-22.
- System reliability analysis: Foundations, pp. 67-85.
- Inference for the exponential life distribution (with F. Proschan), pp. 143-164.
- A guide to the Bayesian approach, pp. 165-168.
- Utility theory, pp. 245-253.

FINAL TECHNICAL REPORT

AIR FORCE AFOSR-81-0122

4/1/81 THROUGH 8/31/86

[redacted]

RESEARCH OF WILLIAM S. JEWELL

Research progress and significant results occurred in three areas of research interest: 1) Reliability Growth and Software Reliability; 2) Bayesian Approximation Methods; and 3) Risk Portfolio Problems. Additionally, work in progress is described under (4) Hierarchical Models. These areas are not mutually exclusive, since many papers are related to each other.

(1) Reliability Growth and Software Reliability

The major contribution in reliability growth modelling was "A General Framework for Learning Curve Reliability Growth Models" (1983 A). Results from this general learning-curve model demonstrate that it is very difficult to make predictions of the ultimate failure rate of a stochastic learning curve from limited-interval initial data, unless a very large number of systems are on test simultaneously, because the data likelihood is very broad. This is true even when the initial failure rate and the learning-curve form are known exactly, and casts doubt upon both classical and Bayesian point estimates of ultimate reliability.

Reliability growth can also occur when discrete defects (out of some finite, but unknown number) are removed during an initial inspection testing program, as in software reliability models. The objective is to estimate the number of defects remaining after the inspection is terminated. "Bayesian

Estimation of Undetected Errors" (1983 B) treats the multiple-inspector, fixed-effort case, where both the error detection efficiencies and the number of bugs are unknown a priori; the full-distributional results also provide a Bayesian generalization to a well-known capture-recapture biometric formula. "Bayesian Extension to a Basic Model of Software Reliability" (1985 C) analyzes the single-inspector continuous-time model, giving a similar Bayesian prediction of the distribution of undetected errors when testing is stopped, as well as an updated estimate of the detection rate parameter. (1985 D) analyzes the dynamics of these estimates when there is a single "probable object" with prior probability, which remains unfound as time progresses; this model is related to international incidents of territorial intrusion.

(2) Bayesian Approximation Methods

For many years I have been interested in linearized approximation to Bayesian predictions, referred to in actuarial articles as "credibility theory." Previous interim reports have described the development of this, by now, rich and varied field.

"Enriched Multinomial Priors Revisited" (1982 E) corrects and updates an earlier paper on the practically important model of the multinomial likelihood with unknown mean vector and precision matrix. The traditional Normal-Wishart prior has the inconvenience of being too "thin" (too few hyperparameters), which also makes the Bayesian mean and covariance predictions too simple, compared to a multi-dimensional credibility approximate forecast. The main result of this paper is a new prior joint distribution for the means and precisions that corrects this thinness.

"Credibility Approximations for Bayesian Prediction of Second Moments" (1984 F) (joint with R. Schneiper) extends the basic model of the credible mean to the problem of approximating the second moments of the predictive distribution as a linear combination of natural first- and second-order statistics; exact results for many important analytic densities also use various combinations of these statistics. Assuming that the various (up to fourth-order) hyperparameters can be estimated, the joint moment forecasts involve the inversion of a 3 by 3 matrix.

(3) Risk Portfolio Problems

Variations of the compound law, which governs the sum of a random number of random variables, are often used to describe an individual risk (insurance contract) which undergoes a random number of random-sized financial shocks in a fixed period, or a risk portfolio composed of such risks. "Approximating the Distribution of a Dynamic Risk Portfolio" (1983 G) is a typical model-development paper in this area that examines the case in which the composition of the portfolio is also random.

Additionally, the exact compound distribution and its variants are notoriously difficult to compute exactly because of the need for high-order convolutions; for many years, approximations based on the normal distribution were the preferred approach. Then, H. Panjer and others discovered that, for a certain class of counting ("frequency" of shocks) distributions, and for discrete and positive shock value ("severity") distributions, one could set up recursive formulae for calculating the distribution of total amount ("loss"). A joint paper with B. Sundt (1981 H) provided extensions to Panjer's result.

Then, in 1984-5, R. Milidiu did his thesis research on the extension to the important case where the "frequency" is Negative Binomial and the "severity" can have both negative and positive values; Panjer-type formulae still exist, but it is now impossible to "get started" on a recursive method. However, various iterative and approximation approaches suggest themselves, and a variety of such strategies were explored computationally. Initial results are reported in the joint paper "Strategies for Computation of Compound Distributions with Two-Sided Severities." (1986 I)

(4) Hierarchical Models

Research effort in 1986 has focussed on hierarchical models, in which "cohort data," generated using different values of the underlying risk parameter, is used to assist the primary prediction process. The necessary correlation between the unknown different parameters is explicated by assuming that they, in turn, depend upon some unknown hyper-prior parameter and distribution, thus giving a heirarchy of random quantities: observables-parameters-hyperparameter. This makes the parameters of the various cohort components exchangeable rvs. In spite of the obvious practical impact of a hierarchical model, few analytic results are known - primarily for the normal-normal-normal (fixed variances) formulation due to Lindley and Smith. The author analyzed the predictive hierarchical mean from the credibility point of view in a 1975 paper.

Based upon this paper and second moment results described in (1984 F), Hans Buhlmann (ETH, Zurich) and I have developed a simultaneous first- and second-moment credibility prediction method, which uses all possible first- and second- order statistics that can be found from cohort data components.

The resulting least-squares analysis can easily be carried out, but the model requires a large number of hyper-moments to be obtained. Asymptotic results, for a large number of data points, or for a large number of cohort components, are of interest, and give insight into how an empirical Bayes estimation of variance should proceed.

Also, to provide an exact analytic formulation against which to test the above computations, the author has been able to generalize the normal-normal-normal model in a heteroscedastic manner, by permitting unknown, but linked, variances at each level of the hierarchy.

Both of these papers will appear shortly.

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85-6	Computational Complexity of Coherent Systems and the Reliability Polynomial	Richard E. Barlow Srinivas Iyer July 1985
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Sheldon M. Ross

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RESEARCH OF SHELDON M. ROSS[redacted]

My research under grant AFOSR-81-0122 has fallen into four main categories: namely (1) Simulation; (2) Software Reliability: Estimation and Testing; (3) Reliability Models; and (4) Peaks from Random Data.

1. Simulation

Almost all dynamic reliability systems can be modelled as Markov processes either in discrete or continuous time and a basic question is to determine the time, starting from a given initial state, until the process enters a state that is considered failed. As such distributions are usually difficult to evaluate analytically, a simulation analysis was presented by Ross and Schechner in [1] "Using Simulation to Estimate First Passage Distributions."

Specifically, they considered a discrete time Markov process $\{X_n, n = 0, 1, \ldots\}$ such that whenever the present state is x the next state is chosen according to the distribution P_x. The initial state i was fixed and for a given set of states A they were interested in estimating the distribution and the mean of N, the number of transitions until the Markov process enters the set A, by use of simulation. By standard techniques such a chain can be simulated until it reaches A --call each such simulation a run. It was then shown that estimators based on

$$N + 1 - \sum_{j=2}^{N+1} P_{X_{j-2}}(A) .$$

where N is the number of steps taken in a given run, and X_j is the j^{th} state in that run and $P_x(A)$ is the probability of going from x to the set A in a single run, has the same mean and smaller variance than the usual estimator N. Hence, the average overall runs of this quantity is a better estimate of $E(N)$ than is the average run size. In addition, a second estimate, based on the observed hazard rate, was given.

Another important problem from a reliability application viewpoint is the estimation of the distribution of the final state. This is important since it represents the failed state and thus repair will depend on it. Such an estimate was presented by working with a modified version of the hazard rate function. Specifically, let $B \subset A$ and define N_B to equal the number of transitions needed to reach B in a run (and thus $N_B = \infty$ if the final state is in $A - B$). Rather than estimating the hazard rate function of N_B, namely $P\{N_B = n \mid N_B \geq n\}$, the modified version $P\{N_B = n \mid N \geq n\}$ was employed, and an estimator based on this was given.

A second research report dealing with simulation was the report [2] (joint with Z. Schechner) entitled "Simulation Uses of the Exponential Distribution." This paper showed how simulated values from an exponential distribution could be effectively used to simulate such diverse quantities as normal order statistics, multi-dimensional Poisson processes and nonhomogeneous Poisson processes.

2. Software Reliability: Estimation and Testing

One of my most significant research accomplishments under the grant has been the development of a model of software reliability and reliability growth.

Consider a complicated system that originally has m defects. Defect i will cause a system failure after a random time that is exponentially distributed with rate λ_i, $i = 1, \ldots, m$. All of the quantities $m, \lambda_1, \ldots, \lambda_m$ are assumed to be unknown. The system is to be run for a time t, with all failures that occur being repaired and the defects that caused the failures being noted. The problem is to estimate the resulting failure rate given that all defects that caused failures in $(0,t)$ are eliminated. Specifically, letting

$$\psi_i(t) = \begin{cases} 1 & \text{if defect } i \text{ does not cause a failure by time } t \\ 0 & \text{otherwise} \end{cases}$$

then we want to estimate

$$\Lambda(t) = \sum_{i=1}^{m} \lambda_i \psi_i(t)$$

In [3] and [4] Ross presented and analyzed the estimator

$$\sum_{i=1}^{D(t)} \frac{e^{-t/T_i}}{T_i(1-e^{-t/T_i})}$$

In addition a stopping rule to enable one to decide when to stop the testing phase and conclude that the remaining error rate is below some preassigned value was developed in [4].

3. Reliability Models

In [5] Ross considered an n component system such that each component is initially on and stays on for a random time at which it fails. The problem of interest is to characterize the distribution of the time until the system fails. Whereas this problem is usually considered under the assumption that the component lives are independent, the model in [5] supposes a Markovian model in which the failure rate of a given component at any time is allowed to depend on the set of working components at that time. Specifically, it supposes that if at some time W, $W \subset \{1, 2, \ldots, n\}$, represents the set of working components then for $i \in W$ the instantaneous failure rate for component i is $\lambda_i(W)$.

Specific conditions that imply that the system life is IFR and IFRA are presented. A method for easily simulating the process is also presented. Finally the model is generalized to allow for the repair of failed components and conditions implying that the process is, in steady state, time reversible are presented.

In [6] Ross and Schechner considered some reliability applications of the variability ordering where if X_1 and X_2 are random variables having respective distributions F_1 and F_2, then we say that $X_1 \underset{v}{\leq} X_2$ (read X_1 is less variable than X_2) if

$$\int_{-\infty}^{\infty} f(x)dF_1(x) \leq \int_{-\infty}^{\infty} f(x)dF_2(x)$$

for all increasing convex functions f.

Applications to a variety of shock and survival models were presented.

In [7] Derman, Lieberman and Ross considered the problem of using replacement to continually extend the life of a system. It was supposed that there was a single vital component which would cause a catastrophe if it failed while in use. By successively determining the times to replace this vital component by one of a finite number of remaining spares the optimal policy was categorized.

4. Peaks in Random Data

In an influential and controversial paper, Raup and Sepkoski ("Periodicity of Extinction in the Geologic Past," Proceedings of the National Academy of Sciences of the U. S., 81, pp. 801-805, 1984) analyzed data relating to the proportion of families that became extinct in each of 39 time periods of (average) length 6.2 million years. They defined an event of mass extinction to have occurred in any time period whose data value exceeded that of its immediate predecessor and follower. Stating that the data indicated a periodicity of mass extinctions, they then presented a statistical analysis which they claim verified the above, and invalidated the previously held belief that such data behaved as a random walk whose incremental change distribution is symmetric about 0.

The statistical analysis of Raup-Sepkoski compared the observed value of a proposed statistic with all 39! possible other values when the data points are permuted. However, as noted by Ross in [8] such a permutation test is only meaningful if the set of alternative hypotheses are such that, conditional on the set of data values, all 39! possible orderings are equally likely. That is, such a test is meaningful if one is testing periodicity against the alternative hypothesis that the data values constitute a random sample from some arbitrary probability distribution. It is not a meaningful test if the alternative is that the incremental changes of the data constitute a random walk. In addition it was then shown in Ross [8] by a nonparametric analysis which employed simulation to test for goodness-of-fit that the random walk model is perfectly consistent with the observed data.

Let $X_1, X_2, \ldots$ be a sequence of random variables and say that a peak occurs at time n if $X_{n-1} < X_n < X_{n+1}$. When the random sequence constitutes a random walk whose incremental change distribution is symmetric about 0 then, as noted in [8], the process of peaks constitutes a renewal process. However, when the X_i constitute a random sample from a continuous distribution then this is no longer true. Indeed, in this situation the times between successive peaks are neither independent nor identically distributed.

The process of peaks, when the data constitutes a random sample from a continuous distribution, is analyzed by Ross in [9]. It is shown that $N(n)$, the number of peaks by time n, is asymptotically normal with mean $(n-1)/3$ and variance $(2n+4)/45$. In addition, it is shown that, with probability 1, $\lim N(n)/n = 1/3$. Finally, it is argued that the proportion of interpeak times that are equal to j converges, with probability 1, to a constant value - call it p_j. The values of the p_j are then given in terms of computable integrals; and in particular it is shown that

$$p_2 = 2/5 \quad p_3 = 1/3 \quad p_4 = 6/35 \quad p_5 = 1/15 \quad p_6 = .02116401 \; .$$

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